

# Package ‘bdsM’

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**Title** Bayesian Dynamic Systems Modeling

**Version** 0.2.1

**Description** Implements methods for building and analyzing models based on panel data as described in the paper by Moral-Benito (2013, <[doi:10.1080/07350015.2013.818003](https://doi.org/10.1080/07350015.2013.818003)>). The package provides functions to estimate dynamic panel data models and analyze the results of the estimation.

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**Author** Mateusz Wyszynski [aut],  
Marcin Dubel [ctb, cre],  
Krzysztof Beck [ctb]

**Maintainer** Marcin Dubel <[marcindubel@gmail.com](mailto:marcindubel@gmail.com)>

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best_models	<i>Table with the best models according to one of the posterior criteria</i>
-------------	--

---

### Description

This function creates a ranking of best models according to one of the possible criterion (PMP under binomial model prior, PMP under binomial-beta model prior,  $R^2$  under binomial model prior,  $R^2$  under binomial-beta model prior). The function gives two types of tables in three different formats: inclusion table (where 1 indicates presence of the regressor in the model and 0 indicates that the variable is excluded from the model) and estimation results table (it displays the best models and estimation output for those models: point estimates, standard errors, significance level, and  $R^2$ ).

### Usage

```
best_models(
  bma_list,
  criterion = 1,
  best = 5,
  round = 3,
```

```

    estimate = TRUE,
    robust = TRUE
  )

```

### Arguments

bma_list	bma object (the result of the bma function)
criterion	The criterion that will be used for a basis of the model ranking: 1 - binomial model prior 2 - binomial-beta model prior
best	The number of the best models to be considered
round	Parameter indicating the decimal place to which number in the tables should be rounded (default round = 3)
estimate	A parameter with values TRUE or FALSE indicating which table should be displayed when TRUE - table with estimation to the results FALSE - table with the inclusion of regressors in the best models
robust	A parameter with values TRUE or FALSE indicating which type of standard errors should be displayed when the function finishes calculations. Works only if estimate = TRUE. Works well when best is small. TRUE - robust standard errors FALSE - regular standard errors

### Value

A list with best\_models objects:

1. matrix with inclusion of the regressors in the best models
2. matrix with estimation output in the best models with regular standard errors
3. matrix with estimation output in the best models with robust standard errors
4. knitr\_kable table with inclusion of the regressors in the best models (the best for the display on the console - up to 11 models)
5. knitr\_kable table with estimation output in the best models with regular standard errors (the best for the display on the console - up to 6 models)
6. knitr\_kable table with estimation output in the best models with robust standard errors (the best for the display on the console - up to 6 models)
7. gTree table with inclusion of the regressors in the best models (displayed as a plot). Use `grid::grid.draw()` to display.

8. gTree table with estimation output in the best models with regular standard errors (displayed as a plot). Use `grid::grid.draw()` to display.
9. gTree table with estimation output in the best models with robust standard errors (displayed as a plot). Use `grid::grid.draw()` to display.

## Examples

```
library(magrittr)

data_prepared <- bdsm::economic_growth[, 1:6] %>%
  bdsm::feature_standardization(
    excluded_cols = c(country, year, gdp)
  ) %>%
  bdsm::feature_standardization(
    group_by_col = year,
    excluded_cols = country,
    scale        = FALSE
  )

bma_results <- bma(
  model_space = bdsm::small_model_space,
  df          = data_prepared,
  round       = 3,
  dilution   = 0
)

best_5_models <- best_models(bma_results, criterion = 1, best = 5, estimate = TRUE, robust = TRUE)
```

---

bma

*Calculation of the bma object*

---

## Description

This function calculates BMA statistics based on the provided model space. Other objects for further analysis are also returned.

## Usage

```
bma(model_space, df, round = 4, EMS = NULL, dilution = 0, dil.Par = 0.5)
```

## Arguments

<code>model_space</code>	List with params and stats from the model space
<code>df</code>	Data frame with data for the SEM analysis.
<code>round</code>	Parameter indicating the decimal place to which number in the BMA tables and prior and posterior model sizes should be rounded (default <code>round = 4</code> )
<code>EMS</code>	Expected model size for model binomial and binomial-beta model prior

dilution	Binary parameter: 0 - NO application of a dilution prior; 1 - application of a dilution prior (George 2010).
dil.Par	Parameter associated with dilution prior - the exponent of the determinant (George 2010). Used only if parameter dilution = 1.

### Value

A list with 16 elements.

**uniform\_table** A table containing the results based on the binomialmodel prior.

**random\_table** A table containing the results based on the binomial-beta model prior.

**reg\_names** A vector containing the names of the regressors, used by the functions.

**R** The total number of regressors.

**num\_of\_models** The number of models present in the model space.

**forJointnes** A table containing model IDs and posterior model probabilities (PMPs) for the jointness function.

**forBestModels** A table containing model IDs, PMPs, coefficients, standard deviations, and standardized regression coefficients (stdRs) for the best\_models function.

**EMS** The expected model size for the binomial and binomial-beta model priors, as specified by the user (default is  $EMS = R/2$ ).

**sizePriors** A table of uniform and random model priors distributed over model sizes for the model\_sizes function.

**PMPs** A table containing the posterior model probabilities for use in the model\_sizes function.

**modelPriors** A table containing the model priors, used by the model\_pmp function.

**dilution** A parameter indicating whether the priors were diluted, used in the model\_sizes function.

**alphas** A vector of coefficients for the lagged dependent variable in the coef\_hist function.

**betas\_nonzero** A vector of nonzero coefficients for the regressors in the coef\_hist function.

**d\_free** A table containing the degrees of freedom for the estimated models in the best\_models function.

**PMStable** A table containing the prior and posterior expected model sizes for the binomial and binomial-beta model priors.

### Examples

```
library(magrittr)

data_prepared <- bdsm::economic_growth[, 1:6] %>%
  bdsm::feature_standardization(
    excluded_cols = c(country, year, gdp)
  ) %>%
  bdsm::feature_standardization(
    group_by_col = year,
    excluded_cols = country,
    scale = FALSE
  )
```

```

bma_results <- bma(
  model_space = bdsm::small_model_space,
  df          = data_prepared,
  round       = 3,
  dilution   = 0
)

```

coef\_hist

*Graphs of the distribution of the coefficients over the model space***Description**

This function draws graphs of the distribution (in the form of histogram or kernel density) of the coefficients for all the considered regressors over the part of the model space that includes this regressors (half of the model space).

**Arguments**

bma_list	bma object (the result of the bma function)
BW	Parameter indicating what method should be chosen to find bin widths for the histograms: <ol style="list-style-type: none"> <li>1. "FD" Freedman-Diaconis method</li> <li>2. "SC" Scott method</li> <li>3. "vec" user specified bin widths provided through a vector (parameter: binW)</li> </ol>
binW	A vector with bin widths to be used to construct histograms for the regressors. The vector must be of the size equal to total number of regressors. The vector with bin widths is used only if parameter BW="vec".
BN	Parameter taking the values (default: BN = 0): <ol style="list-style-type: none"> <li>1 - the histogram will be build based on the number of bins specified by the user through parameter num. If BN=1, the function ignores parameters BW.</li> <li>0 - the histogram will be build in line with parameter BW</li> </ol>
num	A vector with the numbers of bins used to be used to construct histograms for the regressors. The vector must be of the size equal to total number of regressors. The vector with bin widths is used only if parameter BN=1.
kernel	A parameter taking the values (default: kernel = 0): <ol style="list-style-type: none"> <li>1 - the function will build graphs using kernel density for the distribution of coefficients (with kernel=1, the function ignores parameters BW and BN)</li> <li>0 - the function will build regular histogram density for the distribution of coefficients</li> </ol>

**Value**

A list with the graphs of the distribution of coefficients for all the considered regressors.

**Examples**

```
library(magrittr)

data_prepared <- bdsm::economic_growth[, 1:6] %>%
  bdsm::feature_standardization(
    excluded_cols = c(country, year, gdp)
  ) %>%
  bdsm::feature_standardization(
    group_by_col = year,
    excluded_cols = country,
    scale        = FALSE
  )

bma_results <- bma(
  model_space = bdsm::small_model_space,
  df          = data_prepared,
  round      = 3,
  dilution  = 0
)

coef_plots <- coef_hist(bma_results, kernel = 1)
```

---

compute\_model\_space\_stats

*Approximate standard deviations for the models*

---

**Description**

Approximate standard deviations are computed for the models in the given model space. Two versions are computed.

**Usage**

```
compute_model_space_stats(
  df,
  dep_var_col,
  timestamp_col,
  entity_col,
  params,
  exact_value = FALSE,
  model_prior = "uniform",
  cl = NULL
)
```

**Arguments**

df	Data frame with data for the SEM analysis.
dep_var_col	Column with the dependent variable
timestamp_col	The name of the column with timestamps
entity_col	Column with entities (e.g. countries)
params	A matrix (with named rows) with each column corresponding to a model. Each column specifies model parameters. Compare with <a href="#">optim_model_space_params</a>
exact_value	Whether the exact value of the likelihood should be computed (TRUE) or just the proportional part (FALSE). Check <a href="#">sem_likelihood</a> for details.
model_prior	Which model prior to use. For now there are two options: 'uniform' and 'binomial-beta'. Default is 'uniform'.
cl	An optional cluster object. If supplied, the function will use this cluster for parallel processing. If NULL (the default), <code>pbapply::pblapply</code> will run sequentially.

**Value**

Matrix with columns describing likelihood and standard deviations for each model. The first row is the likelihood for the model (computed using the parameters in the provided model space). The second row is almost  $1/2 * BIC_k$  as in Raftery's Bayesian Model Selection in Social Research eq. 19 (see TODO in the code below). The third row is model posterior probability. Then there are rows with standard deviations for each parameter. After that we have rows with robust standard deviation (not sure yet what exactly "robust" means).

**Examples**

```
library(magrittr)
data_prepared <- bdsm::economic_growth[, 1:6] %>%
  bdsm::feature_standardization(
    excluded_cols = c(country, year, gdp)
  ) %>%
  bdsm::feature_standardization(
    group_by_col = year,
    excluded_cols = country,
    scale = FALSE
  )

compute_model_space_stats(
  df = data_prepared,
  dep_var_col = gdp,
  timestamp_col = year,
  entity_col = country,
  params = small_model_space$params
)
```



---

economic\_growth      *Economic Growth Data*

---

### Description

Data used in Growth Empirics in Panel Data under Model Uncertainty and Weak Exogeneity (Moral-Benito, 2016, Journal of Applied Econometrics).

### Usage

economic\_growth

### Format

economic\_growth:

A data frame with 365 rows and 12 columns (73 countries and 4 periods + extra one for lagged dependent variable):

**year** Year

**country** Country ID

**gdp** Logarithm of GDP per capita (2000 US dollars at PP)

**ish** Ratio of real domestic investment to GDP

**sed** Stock of years of secondary education in the total population

**pgrw** Average growth rate of population

**pop** Population in millions of people

**ipr** Purchasing-power-parity numbers for investment goods

**opem** Exports plus imports as a share of GDP

**gsh** Ratio of government consumption to GDP

**lnlex** Logarithm of the life expectancy at birth

**polity** Composite index given by the democracy score minus the autocracy score

### Source

<http://qed.econ.queensu.ca/jae/datasets/moral-benito001/>

---

exogenous\_matrix      *Matrix with exogenous variables for SEM representation*

---

### Description

Create matrix which contains exogenous variables used in the Simultaneous Equations Model (SEM) representation. Currently these are: dependent variable from the lowest time stamp and regressors from the second lowest time stamp. The matrix is then used to compute likelihood for SEM analysis.

**Usage**

```
exogenous_matrix(df, timestamp_col, entity_col, dep_var_col)
```

**Arguments**

df	Data frame with data for the SEM analysis.
timestamp_col	Column which determines time periods. For now only natural numbers can be used as timestamps
entity_col	Column which determines entities (e.g. countries, people)
dep_var_col	Column with dependent variable

**Value**

Matrix of size  $N \times k+1$  where  $N$  is the number of entities considered and  $k$  is the number of chosen regressors

**Examples**

```
set.seed(1)
df <- data.frame(
  entities = rep(1:4, 5),
  times = rep(seq(1960, 2000, 10), each = 4),
  dep_var = stats::rnorm(20), a = stats::rnorm(20), b = stats::rnorm(20)
)
exogenous_matrix(df, times, entities, dep_var)
```

---

feature\_standardization

*Perform feature standardization*

---

**Description**

This function performs **feature standardization** (also known as z-score normalization) by centering the features around their mean and scaling by their standard deviation.

**Usage**

```
feature_standardization(df, excluded_cols, group_by_col, scale = TRUE)
```

**Arguments**

df	Data frame with the data.
excluded_cols	Unquoted column names to exclude from standardization. If missing, all columns are standardized.
group_by_col	Unquoted column names to group the data by before applying standardization. If missing, no grouping is performed.
scale	Logical. If TRUE (default) scales by the standard deviation.

**Value**

A data frame with standardized features.

**Examples**

```
df <- data.frame(
  year = c(2000, 2001, 2002, 2003, 2004),
  country = c("A", "A", "B", "B", "C"),
  gdp = c(1, 2, 3, 4, 5),
  ish = c(2, 3, 4, 5, 6),
  sed = c(3, 4, 5, 6, 7)
)

# Standardize every column
df_with_only_numeric_values <- df[, setdiff(names(df), "country")]
feature_standardization(df_with_only_numeric_values)

# Standardize all columns except 'country'
feature_standardization(df, excluded_cols = country)

# Standardize across countries (grouped by 'country')
feature_standardization(df, group_by_col = country)

# Standardize, excluding 'country' and group-wise by 'year'
feature_standardization(df, excluded_cols = country, group_by_col = year)
```

---

full\_model\_space

*Example output of the optim\_model\_space function*

---

**Description**

A list with two elements: params and stats computed using the optim\_model\_space function and the economic\_growth dataset.

**Usage**

```
full_model_space
```

**Format**

full\_model\_space:

A list with two elements.

**params** A double matrix with 40 rows and  $2^9 = 512$  columns with the parameters for the model space. Each column represents a different model.

**stats** A matrix representing the statistics computed with compute\_model\_space\_stats based on params. The first row contains likelihoods for the models. The second row are almost  $1/2 * BIC_k$  as in Raftery's Bayesian Model Selection in Social Research, eq. 19. The rows 3-7 are standard deviations. Finally, the rows 8-12 are robust standard deviations.

hessian *Hessian matrix*

---

**Description**

Creates the hessian matrix for a given likelihood function.

**Usage**

```
hessian(lik, theta, ...)
```

**Arguments**

lik	function
theta	kx1 matrix
...	other parameters passed to lik function.

**Value**

Hessian kxk matrix where k is the number of parameters included in the theta matrix

**Examples**

```
lik <- function(theta) {  
  return(theta[1]^2 + theta[2]^2)  
}  
  
hessian(lik, c(1, 1))
```

---

init\_model\_space\_params  
*Initialize model space matrix*

---

**Description**

This function builds a representation of the model space, by creating a dataframe where each column represents values of the parameters for a given model. Real value means that the parameter is included in the model. A parameter not present in the model is marked as NA.

**Usage**

```
init_model_space_params(
  df,
  timestamp_col,
  entity_col,
  dep_var_col,
  init_value = 1
)
```

**Arguments**

df	Data frame with data for the SEM analysis.
timestamp_col	Column which determines time periods. For now only natural numbers can be used as timestamps
entity_col	Column which determines entities (e.g. countries, people)
dep_var_col	Column with dependent variable
init_value	Initial value for parameters present in the model. Default is 1.

**Details**

Currently the set of features is assumed to be all columns which remain after excluding `timestamp_col`, `entity_col` and `dep_var_col`.

A power set of all possible exclusions of linear dependence on the given feature is created, i.e. if there are 4 features we end up with  $2^4$  possible models (for each model we independently decide whether to include or not a feature).

**Value**

matrix of model parameters

**Examples**

```
library(magrittr)

data_prepared <- bdsm::economic_growth[, 1:5] %>%
  bdsm::feature_standardization(
    excluded_cols = c(country, year, gdp)
  ) %>%
  bdsm::feature_standardization(
    group_by_col = year,
    excluded_cols = country,
    scale        = FALSE
  )

init_model_space_params(data_prepared, year, country, gdp)
```

jointness

*Calculation of of the jointness measures***Description**

This function calculates four types of the jointness measures based on the posterior model probabilities calculated using binomial and binomial-beta model prior. The four measures are:

1. HCGHM - for Hofmarcher et al. (2018) measure;
2. LS - for Ley & Steel (2007) measure;
3. DW - for Doppelhofer & Weeks (2009) measure;
4. PPI - for posterior probability of including both variables.

The measures under binomial model prior will appear in a table above the diagonal, and the measure calculated under binomial-beta model prior below the diagonal.

**REFERENCES**

- Doppelhofer G, Weeks M (2009) Jointness of growth determinants. *Journal of Applied Econometrics.*, 24(2), 209-244. doi: 10.1002/jae.1046
- Hofmarcher P, Crespo Cuaresma J, Grün B, Humer S, Moser M (2018) Bivariate jointness measures in Bayesian Model Averaging: Solving the conundrum. *Journal of Macroeconomics*, 57, 150-165. doi: 10.1016/j.jmacro.2018.05.005
- Ley E, Steel M (2007) Jointness in Bayesian variable selection with applications to growth regression. *Journal of Macroeconomics*, 29(3), 476-493. doi: 10.1016/j.jmacro.2006.12.002

**Usage**

```
jointness(bma_list, measure = "HCGHM", rho = 0.5, round = 3)
```

**Arguments**

bma_list	bma object (the result of the bma function)
measure	Parameter for choosing the measure of jointness: HCGHM - for Hofmarcher et al. (2018) measure; LS - for Ley & Steel (2007) measure; DW - for Doppelhofer & Weeks (2009) measure; PPI - for posterior probability of including both variables.
rho	The parameter "rho" ( $\rho$ ) to be used in HCGHM jointness measure (default rho = 0.5). Works only if HCGHM measure is chosen (Hofmarcher et al. 2018).
round	Parameter indicating the decimal place to which the jointness measures should be rounded (default round = 3).

**Value**

A table with jointness measures for all the pairs of regressors used in the analysis. Parameter "above" indicates what model prior is used for the values ABOVE the diagonal, and parameter "below" indicates what model prior is used for the values BELOW the diagonal.

**Examples**

```
library(magrittr)

data_prepared <- bdsm::economic_growth[, 1:6] %>%
  bdsm::feature_standardization(
    excluded_cols = c(country, year, gdp)
  ) %>%
  bdsm::feature_standardization(
    group_by_col = year,
    excluded_cols = country,
    scale        = FALSE
  )

bma_results <- bma(
  model_space = bdsm::small_model_space,
  df          = data_prepared,
  round       = 3,
  dilution   = 0
)

jointness_table <- jointness(bma_results, measure = "HCGHM", rho = 0.5, round = 3)
```

---

join_lagged_col	<i>Dataframe with no lagged column</i>
-----------------	--

---

**Description**

This function allows to turn data in the format with lagged values for a chosen column (i.e. there are two columns with the same quantity, but one column is lagged in time) into the format with just one column

**Usage**

```
join_lagged_col(
  df,
  col,
  col_lagged,
  timestamp_col,
  entity_col,
  timestep = NULL
)
```

**Arguments**

df	Dataframe with data with a column with lagged values
col	Column with quantity not lagged
col_lagged	Column with the same quantity as col, but the values are lagged in time
timestamp_col	Column with timestamps (e.g. years)
entity_col	Column with entities (e.g. countries)
timestep	Difference between timestamps (e.g. 10)

**Value**

A dataframe with two columns merged, i.e. just one column with the desired quantity is left.

**Examples**

```
df <- data.frame(  
  year = c(2000, 2001, 2002, 2003, 2004),  
  country = c("A", "A", "B", "B", "C"),  
  gdp = c(1, 2, 3, 4, 5),  
  gdp_lagged = c(NA, 1, 2, 3, 4)  
)  
  
join_lagged_col(df, gdp, gdp_lagged, year, country, 1)
```

---

matrices\_from\_df      *List of matrices for SEM model*

---

**Description**

List of matrices for SEM model

**Usage**

```
matrices_from_df(  
  df,  
  timestamp_col,  
  entity_col,  
  dep_var_col,  
  lin_related_regressors = NULL,  
  which_matrices = c("Y1", "Y2", "Z", "cur_Y2", "cur_Z", "res_maker_matrix")  
)
```



**Arguments**

<code>df</code>	Dataframe with data for the likelihood computations.
<code>timestamp_col</code>	Column which determines time stamps. For now only natural numbers can be used.
<code>entity_col</code>	Column which determines entities (e.g. countries, people)
<code>dep_var_col</code>	Column with dependent variable
<code>lin_related_regressors</code>	Vector of strings of column names. Which subset of regressors is in non trivial linear relation with the dependent variable ( <code>dep_var_col</code> ). In other words regressors with non-zero beta parameters.
<code>which_matrices</code>	character vector with names of matrices which should be computed. Possible matrices are "Y1", "Y2", "Z", "cur_Y2", "cur_Z", "res_maker_matrix". Default is <code>c("Y1", "Y2", "Z", "cur_Y2", "cur_Z", "res_maker_matrix")</code> in which case all possible matrices are generated

**Value**

Named list with matrices as its elements

**Examples**

```
matrices_from_df(economic_growth, year, country, gdp, c("pop", "sed"),
                 c("Y1", "Y2"))
```

---

<code>model_pmp</code>	<i>Graphs of the prior and posterior model probabilities for the best individual models</i>
------------------------	---

---

**Description**

This function draws four graphs of prior and posterior model probabilities for the best individual models:

- a) The results with binomial model prior (based on PMP - posterior model probability)
- b) The results with binomial-beta model prior (based on PMP - posterior model probability)

Models on the graph are ordered according to their posterior model probability.

**Arguments**

<code>bma_list</code>	<code>bma_list</code> object (the result of the <code>bma</code> function)
<code>top</code>	The number of the best model to be placed on the graphs

**Value**

A list with three graphs with prior and posterior model probabilities for individual models:

1. The results with binomial model prior (based on PMP - posterior model probability)
2. The results with binomial-beta model prior (based on PMP - posterior model probability)
3. On graph combining the aforementioned graphs

**Examples**

```
library(magrittr)

data_prepared <- bdsm::economic_growth[, 1:6] %>%
  bdsm::feature_standardization(
    excluded_cols = c(country, year, gdp)
  ) %>%
  bdsm::feature_standardization(
    group_by_col = year,
    excluded_cols = country,
    scale       = FALSE
  )

bma_results <- bma(
  model_space = bdsm::small_model_space,
  df          = data_prepared,
  round       = 3,
  dilution   = 0
)

model_graphs <- model_pmp(bma_results, top = 16)
```

---

model\_sizes

*Graphs of the prior and posterior model probabilities of the model sizes*

---

**Description**

This function draws four graphs of prior and posterior model probabilities:

- a) The results with binomial model prior (based on PMP - posterior model probability)
- b) The results with binomial-beta model prior (based on PMP - posterior model probability)

**Arguments**

`bma_list`            `bma_list` object (the result of the `bma` function)

**Value**

A list with three graphs with prior and posterior model probabilities for model sizes:

1. The results with binomial model prior (based on PMP - posterior model probability)
2. The results with binomial-beta model prior (based on PMP - posterior model probability)
3. One graph combining all the aforementioned graphs

**Examples**

```
library(magrittr)

data_prepared <- bdsm::economic_growth[, 1:6] %>%
  bdsm::feature_standardization(
    excluded_cols = c(country, year, gdp)
  ) %>%
  bdsm::feature_standardization(
    group_by_col = year,
    excluded_cols = country,
    scale        = FALSE
  )

bma_results <- bma(
  model_space = bdsm::small_model_space,
  df          = data_prepared,
  round       = 3,
  dilution   = 0
)

size_graphs <- model_sizes(bma_results)
```

---

optim\_model\_space      *Calculation of the model\_space object*

---

**Description**

This function calculates model space, values of the maximized likelihood function, BICs, and standard deviations of the parameters that will be used in Bayesian model averaging.

**Usage**

```
optim_model_space(
  df,
  timestamp_col,
```

```

entity_col,
dep_var_col,
init_value,
exact_value = FALSE,
cl = NULL,
control = list(trace = 2, maxit = 10000, fnscale = -1, REPORT = 100, scale = 0.05)
)

```

### Arguments

df	Data frame with data for the analysis.
timestamp_col	The name of the column with time stamps
entity_col	Column with entities (e.g. countries)
dep_var_col	Column with the dependent variable
init_value	The value with which the model space will be initialized. This will be the starting point for the numerical optimization.
exact_value	Whether the exact value of the likelihood should be computed (TRUE) or just the proportional part (FALSE). Check <a href="#">sem_likelihood</a> for details.
cl	An optional cluster object. If supplied, the function will use this cluster for parallel processing. If NULL (the default), <code>pbapply::pblapply</code> will run sequentially.
control	a list of control parameters for the optimization which are passed to <a href="#">optim</a> . Default is <code>list(trace = 2, maxit = 10000, fnscale = -1, REPORT = 100, scale = 0.05)</code> , but note that <code>scale</code> is used only for adjusting the <code>parscale</code> element added later in the function code.

### Value

List with two objects:

1. `params` - table with parameters of all estimated models
2. `stats` - table with the value of maximized likelihood function, BIC, and standard errors for all estimated models

### Examples

```

## Not run:
library(magrittr)

data_prepared <- bdsm::economic_growth[, 1:5] %>%
  bdsm::feature_standardization(
    excluded_cols = c(country, year, gdp)
  ) %>%
  bdsm::feature_standardization(
    group_by_col = year,
    excluded_cols = country,

```

```

    scale      = FALSE
  )

optim_model_space(
  df          = data_prepared,
  dep_var_col = gdp,
  timestamp_col = year,
  entity_col  = country,
  init_value  = 0.5
)

## End(Not run)

```

---

optim\_model\_space\_params

*Finds MLE parameters for each model in the given model space*

---

## Description

Given a dataset and an initial value for parameters, initializes a model space with parameters equal to the initial value for each model. Then for each model performs a numerical optimization and finds parameters which maximize the likelihood.

## Usage

```

optim_model_space_params(
  df,
  timestamp_col,
  entity_col,
  dep_var_col,
  init_value,
  exact_value = FALSE,
  cl = NULL,
  control = list(trace = 2, maxit = 10000, fnscale = -1, REPORT = 100, scale = 0.05)
)

```

## Arguments

df	Data frame with data for the analysis.
timestamp_col	The name of the column with time stamps.
entity_col	Column with entities (e.g. countries).
dep_var_col	Column with the dependent variable.
init_value	The value with which the model space will be initialized. This will be the starting point for the numerical optimization.
exact_value	Whether the exact value of the likelihood should be computed (TRUE) or just the proportional part (FALSE). Check <a href="#">sem_likelihood</a> for details.

- cl** An optional cluster object. If supplied, the function will use this cluster for parallel processing. If NULL (the default), `pbapply::pblapply` will run sequentially.
- control** a list of control parameters for the optimization which are passed to `optim`. Default is `list(trace = 2, maxit = 10000, fnscale = -1, REPORT = 100, scale = 0.05)`.

### Value

List (or matrix) of parameters describing analyzed models.

---

original\_economic\_growth

*Economic Growth Data in the original format*

---

### Description

Data used in Growth Empirics in Panel Data under Model Uncertainty and Weak Exogeneity (Moral-Benito, 2016, Journal of Applied Econometrics).

### Usage

original\_economic\_growth

### Format

original\_economic\_growth:

A data frame with 292 rows and 13 columns (73 countries and 4 periods + extra one for lagged dependent variable):

**year** Year

**country** Country ID

**gdp** Logarithm of GDP per capita (2000 US dollars at PP)

**gdp\_lag** Lagged logarithm of GDP per capita (2000 US dollars at PP)

**ish** Ratio of real domestic investment to GDP

**sed** Stock of years of secondary education in the total population

**pgrw** Average growth rate of population

**pop** Population in millions of people

**ipr** Purchasing-power-parity numbers for investment goods

**opem** Exports plus imports as a share of GDP

**gsh** Ratio of government consumption to GDP

**lnlex** Logarithm of the life expectancy at birth

**polity** Composite index given by the democracy score minus the autocracy score

### Source

<http://qed.econ.queensu.ca/jae/datasets/moral-benito001/>

---

`regressor_names_from_params_vector`*Helper function to extract names from a vector defining a model*

---

**Description**

For now it is assumed that we can only exclude linear relationships between regressors and the dependent variable.

**Usage**

```
regressor_names_from_params_vector(params)
```

**Arguments**

`params` a vector with parameters describing the model

**Details**

The vector needs to have named rows, i.e. it is assumed it comes from a model space (see [init\\_model\\_space\\_params](#) for details).

**Value**

Names of regressors which are assumed to be linearly connected with dependent variable within the model described by the `params` vector.

**Examples**

```
params <- c(alpha = 1, beta_gdp = 1, beta_gdp_lagged = 1, phi_0 = 1, err_var = 1)
regressor_names_from_params_vector(params)
```

---

`residual_maker_matrix` *Residual Maker Matrix*

---

**Description**

Create residual maker matrix from a given matrix `m`. See article about [projection matrix](#) on the Wikipedia.

**Usage**

```
residual_maker_matrix(m)
```

**Arguments**

m                    Matrix

**Value**

M x M matrix where M is the number of rows in the m matrix.

**Examples**

```
residual_maker_matrix(matrix(c(1,2,3,4), nrow = 2))
```

---

sem\_B\_matrix                    *Coefficients matrix for SEM representation*

---

**Description**

Create coefficients matrix for Simultaneous Equations Model (SEM) representation.

**Usage**

```
sem_B_matrix(alpha, periods_n, beta = c())
```

**Arguments**

alpha                numeric  
periods\_n            integer  
beta                  numeric vector. Default is c() for no regressors case.

**Value**

List with two matrices B11 and B12

**Examples**

```
sem_B_matrix(3, 4, 4:6)
```



---

sem_C_matrix	<i>Coefficients matrix for initial conditions</i>
--------------	---

---

**Description**

Create matrix for Simultaneous Equations Model (SEM) representation with coefficients placed next to initial values of regressors, dependent variable and country-specific time-invariant variables.

**Usage**

```
sem_C_matrix(alpha, phi_0, periods_n, beta = c(), phi_1 = c())
```

**Arguments**

alpha	numeric
phi_0	numeric
periods_n	numeric
beta	numeric vector. Default is c() for no regressors case.
phi_1	numeric vector. Default is c() for no regressors case.

**Value**

matrix

**Examples**

```
alpha <- 9
phi_0 <- 19
beta <- 11:15
phi_1 <- 21:25
periods_n <- 4
sem_C_matrix(alpha, phi_0, periods_n, beta, phi_1)
```

---

sem_dep_var_matrix	<i>Matrix with dependent variable data for SEM representation</i>
--------------------	---

---

**Description**

Create matrix which contains dependent variable data used in the Simultaneous Equations Model (SEM) representation on the left hand side of the equations. The matrix contains the data for time periods greater than or equal to the second lowest time stamp. The matrix is then used to compute likelihood for SEM analysis.

**Usage**

```
sem_dep_var_matrix(df, timestamp_col, entity_col, dep_var_col)
```

**Arguments**

df	Data frame with data for the SEM analysis.
timestamp_col	Column which determines time periods. For now only natural numbers can be used as timestamps
entity_col	Column which determines entities (e.g. countries, people)
dep_var_col	Column with dependent variable

**Value**

Matrix of size  $N \times T$  where  $N$  is the number of entities considered and  $T$  is the number of periods greater than or equal to the second lowest time stamp.

**Examples**

```
set.seed(1)
df <- data.frame(
  entities = rep(1:4, 5),
  times = rep(seq(1960, 2000, 10), each = 4),
  dep_var = stats::rnorm(20), a = stats::rnorm(20), b = stats::rnorm(20)
)
sem_dep_var_matrix(df, times, entities, dep_var)
```

---

sem\_likelihood

*Likelihood for the SEM model*

---

**Description**

Likelihood for the SEM model

**Usage**

```
sem_likelihood(
  params,
  data,
  timestamp_col,
  entity_col,
  dep_var_col,
  lin_related_regressors = NULL,
  per_entity = FALSE,
  exact_value = TRUE
)
```

**Arguments**

params	Parameters describing the model. Can be either a vector or a list with named parameters. See 'Details'
data	Data for the likelihood computations. Can be either a list of matrices or a dataframe. If the dataframe, additional parameters are required to build the matrices within the function.
timestamp_col	Column which determines time stamps. For now only natural numbers can be used.
entity_col	Column which determines entities (e.g. countries, people)
dep_var_col	Column with dependent variable
lin_related_regressors	Which subset of columns should be used as regressors for the current model. In other words regressors are the total set of regressors and lin_related_regressors are the ones for which linear relation is not set to zero for a given model.
per_entity	Whether to compute overall likelihood or a vector of likelihoods with per entity value
exact_value	Whether the exact value of the likelihood should be computed (TRUE) or just the proportional part (FALSE). Currently TRUE adds: 1. a normalization constant coming from Gaussian distribution, 2. a term disappearing during likelihood simplification in Likelihood-based Estimation of Dynamic Panels with Predetermined Regressors by Moral-Benito (see Appendix A.1). The latter happens when transitioning from equation (47) to equation (48), in step 2: the term $\text{trace}(\text{HG}_{22})$ is dropped, because it can be assumed to be constant from Moral-Benito perspective. To get the exact value of the likelihood we have to take this term into account.

**Details**

The params argument is a list that should contain the following components:

alpha scalar value which determines linear dependence on lagged dependent variable

phi\_0 scalar value which determines linear dependence on the value of dependent variable at the lowest time stamp

err\_var scalar value which determines classical error component ( $\Sigma_{11}$  matrix,  $\sigma_{\epsilon}^2$ )

dep\_vars double vector of length equal to the number of time stamps (i.e. time stamps greater than or equal to the second lowest time stamp)

beta double vector which determines the linear dependence on regressors different than the lagged dependent variable; The vector should have length equal to the number of regressors.

phi\_1 double vector which determines the linear dependence on initial values of regressors different than the lagged dependent variable; The vector should have length equal to the number of regressors.

phis double vector which together with psis determines upper right and bottom left part of the covariance matrix; The vector should have length equal to the number of regressors times number of time stamps minus 1, i.e.  $\text{regressors}_n * (\text{periods}_n - 1)$

psis double vector which together with phis determines upper right and bottom left part of the covariance matrix; The vector should have length equal to the number of regressors times number of

time stamps minus 1 times number of time stamps divided by 2, i.e.  $\text{regressors\_n} * (\text{periods\_n} - 1) * \text{periods\_n} / 2$

### Value

The value of the likelihood for SEM model (or a part of interest of the likelihood)

### Examples

```
set.seed(1)
df <- data.frame(
  entities = rep(1:4, 5),
  times = rep(seq(1960, 2000, 10), each = 4),
  dep_var = stats::rnorm(20), a = stats::rnorm(20), b = stats::rnorm(20)
)
df <-
  feature_standardization(df, excluded_cols = c(times, entities))
sem_likelihood(0.5, df, times, entities, dep_var)
```

---

sem\_psi\_matrix

*Matrix with psi parameters for SEM representation*

---

### Description

Matrix with psi parameters for SEM representation

### Usage

```
sem_psi_matrix(psis, timestamps_n, features_n)
```

### Arguments

psis            double vector with psi parameter values  
 timestamps\_n   number of time stamps (e.g. years)  
 features\_n     number of features (e.g. population size, investment rate)

### Value

A matrix with `timestamps_n` rows and  $(\text{timestamps\_n} - 1) * \text{feature\_n}$  columns. Psis are filled in row by row in a block manner, i.e. blocks of size `feature_n` are placed next to each other

### Examples

```
sem_psi_matrix(1:30, 4, 5)
```

---

sem\_regressors\_matrix *Matrix with regressors data for SEM representation*

---

## Description

Create matrix which contains regressors data used in the Simultaneous Equations Model (SEM) representation on the left hand side of the equations. The matrix contains regressors data for time periods greater than or equal to the second lowest time stamp. The matrix is then used to compute likelihood for SEM analysis.

## Usage

```
sem_regressors_matrix(df, timestamp_col, entity_col, dep_var_col)
```

## Arguments

df	Data frame with data for the SEM analysis.
timestamp_col	Column which determines time periods. For now only natural numbers can be used as timestamps
entity_col	Column which determines entities (e.g. countries, people)
dep_var_col	Column with dependent variable

## Value

Matrix of size  $N \times (T-1) \times k$  where  $N$  is the number of entities considered,  $T$  is the number of periods greater than or equal to the second lowest time stamp and  $k$  is the number of chosen regressors. If there are no regressors returns NULL.

## Examples

```
set.seed(1)
df <- data.frame(
  entities = rep(1:4, 5),
  times = rep(seq(1960, 2000, 10), each = 4),
  dep_var = stats::rnorm(20), a = stats::rnorm(20), b = stats::rnorm(20)
)
sem_regressors_matrix(df, times, entities, dep_var)
```

---

sem_sigma_matrix	<i>Covariance matrix for SEM representation</i>
------------------	---

---

**Description**

Create covariance matrix for Simultaneous Equations Model (SEM) representation. Only the part necessary to compute concentrated likelihood function is computed (cf. Appendix in the Moral-Benito paper)

**Usage**

```
sem_sigma_matrix(err_var, dep_vars, phis = c(), psis = c())
```

**Arguments**

err_var	numeric
dep_vars	numeric vector
phis	numeric vector
psis	numeric vector

**Value**

List with two matrices Sigma11 and Sigma12

**Examples**

```
err_var <- 1
dep_vars <- c(2, 2, 2, 2)
phis <- c(10, 10, 20, 20, 30, 30)
psis <- c(101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112)
sem_sigma_matrix(err_var, dep_vars, phis, psis)
```

---

small_model_space	<i>Example output of the optim_model_space function (small version)</i>
-------------------	---

---

**Description**

A list with two elements: params and stats computed using the optim\_model\_space function and the economic\_growth dataset, but using only three regressors: ish, sed and pgrw.

**Usage**

```
small_model_space
```

**Format**

`small_model_space`:

A list with two elements.

**params** A double matrix with 40 rows and  $2^3 = 8$  columns with the parameters for the model space. Each column represents a different model.

**stats** A matrix representing the statistics computed with `compute_model_space_stats` based on `params`. The first row contains likelihoods for the models. The second row are almost  $1/2 * \text{BIC}_k$  as in Raftery's Bayesian Model Selection in Social Research, eq. 19. The rows 3-7 are standard deviations. Finally, the rows 8-12 are robust standard deviations.

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